

# Heavy neutrino impact on the triple Higgs coupling

J. Baglio<sup>1,\*</sup> and C. Weiland<sup>2,†</sup>

<sup>1</sup>*Institut für Theoretische Physik, Eberhard Karls Universität Tübingen,  
Auf der Morgenstelle 14, D-72076 Tübingen, Germany.*

<sup>2</sup>*Institute for Particle Physics Phenomenology, Department of Physics,  
Durham University, South Road, Durham DH1 3LE, United Kingdom.*

We present the first calculation of the one-loop corrections to the triple Higgs coupling in the framework of a simplified 3+1 Dirac neutrino model, that is three light neutrinos plus one heavy neutrino embedded in the Standard Model (SM). The triple Higgs coupling is a key parameter of the scalar potential triggering the electroweak symmetry-breaking mechanism in the SM. The impact of the heavy neutrino can be as large as +20% to +30% for parameter points allowed by the current experimental constraints depending on the tightness of the perturbative bound. This can be probed at the high-luminosity LHC, at future electron-positron colliders and at the Future Circular Collider in hadron-hadron mode, an envisioned 100 TeV  $pp$  machine. Our calculation, being done in the mass basis, can be extended to any model using the neutrino portal. In addition, the effects that we have calculated are expected to be enhanced if additional heavy fermions with large Yukawa couplings are included, as in low-scale seesaw mechanisms.

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## I. INTRODUCTION

The biggest highlight of the 7–8 TeV run of the Large Hadron Collider (LHC) at CERN is the discovery in 2012 of a Higgs boson with a mass of around 125 GeV [1, 2]. The Higgs boson is the remnant of the electroweak symmetry-breaking (EWSB) mechanism [3–6] that gives their masses to the other fundamental particles and unitarizes the scattering of weak bosons [7, 8]. While data agrees well with Standard Model (SM) expectations, open questions remain and the measure of the triple Higgs coupling would ultimately test the electroweak sector of the SM by allowing the reconstruction of the scalar potential. It is one of the main goals of the high-luminosity run of the LHC (HL-LHC) and of future colliders, as it is central to ongoing case studies; for reviews, see e.g. Refs. [9, 10].

Neutrino oscillations form the only confirmed phenomenon that absolutely calls for a particle physics explanation [11] and an extension of the SM. One of the simplest possibilities to explain the nonzero neutrino masses and mixing is to add fermionic gauge singlets that will play the role of right-handed neutrinos. The addition of these heavy sterile neutrinos is quite generic, appearing in the type I seesaw [12–18] and its variants [19–27], for example. In particular, these heavy sterile neutrinos behave like pseudo-Dirac fermions in low-scale seesaw mechanisms that rely on an approximately conserved lepton number. Here, we will consider a simplified 3+1 model where the SM is phenomenologically modified to account for 3 light massive neutrinos and one heavy sterile neutrino, all of them being Dirac fermions. This will capture the beyond-the-SM (BSM) effects that we want

to analyze, effects that we expect to arise in all models where the neutrino portal connects the SM with new physics be it dark matter or hidden sectors.

In this article we study the impact of this extended neutrino sector on the Higgs sector and in particular on the triple Higgs coupling, proposing the latter as a new observable for neutrino physics and showing that it can constrain heavy neutrinos in a mass regime difficult to probe otherwise. We calculate for the first time the full one-loop corrections to the triple Higgs coupling in a simplified 3+1 model with three light Dirac neutrinos, identified with those of the SM, plus one heavy Dirac neutrino. We describe the analytical setup of our calculation, the model as well as the theoretical and experimental constraints considered before presenting the numerical results. For parameter points that are allowed by the constraints we find effects that can be probed at the HL-LHC, at future electron-positron colliders and at the Future Circular Collider in hadron-hadron mode (FCC-hh), a potential 100 TeV  $pp$  collider following the LHC. These large deviations are entirely due to quantum corrections, contrarily to supersymmetric or composite models where tree-level corrections are dominant, leading to a new type of scenario for the study of the triple Higgs coupling. Moreover, larger deviations are expected in UV complete models where more heavy neutrinos are included, as in the case of low-scale seesaw mechanisms.

## II. CALCULATION SETUP

We start with the SM scalar potential,

$$V(\Phi) = -\mu^2|\Phi|^2 + \lambda|\Phi|^4, \quad (1)$$

with

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}G^+ \\ v + H + iG^0 \end{pmatrix}. \quad (2)$$

\* [julien.baglio@uni-tuebingen.de](mailto:julien.baglio@uni-tuebingen.de)

† [cedric.weiland@durham.ac.uk](mailto:cedric.weiland@durham.ac.uk)

Here,  $H$  is the Higgs boson,  $G^0$  the neutral Goldstone boson,  $G^\pm$  the charged Goldstone boson and  $v \simeq 246$  GeV the vacuum expectation value. We can define the Higgs tadpole  $t_H$ , the Higgs mass  $M_H$  and the triple Higgs coupling  $\lambda_{HHH}$  as the linear, quadratic and cubic term of the scalar potential, respectively, in terms of the field  $H$ . At tree level,  $t_H = 0$  and the triple Higgs coupling is

$$\lambda^0 = -\frac{3M_H^2}{v}. \quad (3)$$

For the one-loop corrections to the triple Higgs coupling, our set of input parameters that need to be renormalized in the on-shell (OS) scheme is the following:

$$M_H, M_W, M_Z, e, t_H. \quad (4)$$

We require that we have no tadpoles at one loop:

$$t_H^{(1)} + \delta t_H = 0 \Rightarrow \delta t_H = -t_H^{(1)}, \quad (5)$$

with  $t_H^{(1)}$  being the one-loop diagrams for  $t_H$ . We also renormalize the Higgs wave function in the OS scheme. The full renormalized one-loop triple Higgs coupling is then  $\lambda_{HHH}^{1r} = \lambda^0 + \lambda_{HHH}^{(1)} + \delta\lambda_{HHH}$  with

$$\begin{aligned} \frac{\delta\lambda_{HHH}}{\lambda^0} = & \frac{3}{2}\delta Z_H + \delta t_H \frac{e}{2M_W \sin\theta_W M_H^2} + \delta Z_e + \frac{\delta M_H^2}{M_H^2} \\ & - \frac{\delta M_W^2}{2M_W^2} + \frac{1}{2} \frac{\cos^2\theta_W}{\sin^2\theta_W} \left( \frac{\delta M_W^2}{M_W^2} - \frac{\delta M_Z^2}{M_Z^2} \right), \end{aligned} \quad (6)$$

where  $\delta Z_H, \delta Z_e, \delta M_H^2, \delta M_W^2, \delta M_Z^2$  are the counterterms of the Higgs fields, the electrical charge, the Higgs mass and the  $W^\pm/Z$  weak boson masses, respectively,  $\theta_W$  is the weak mixing angle and  $\lambda_{HHH}^{(1)}$  stands for the one-loop diagrams of the process  $H^* \rightarrow HH$ . We define, for the analysis of the results,

$$\begin{aligned} \Delta^{(1)}\lambda_{HHH} &= \frac{1}{\lambda^0} (\lambda_{HHH}^{1r} - \lambda^0), \\ \Delta^{\text{BSM}} &= \frac{1}{\lambda_{HHH}^{1r, \text{SM}}} (\lambda_{HHH}^{1r, \text{full}} - \lambda_{HHH}^{1r, \text{SM}}). \end{aligned} \quad (7)$$

In order to be independent of the light fermion masses, we use the following condition for the electric charge renormalization [28, 29],

$$\delta Z_e = \frac{\sin\theta_W}{\cos\theta_W} \frac{\text{Re}\Sigma_{\gamma Z}^T(0)}{M_Z^2} - \frac{\text{Re}\Sigma_{\gamma\gamma}^T(M_Z^2)}{M_Z^2}, \quad (8)$$

where  $\Sigma_{XY}$  stands for the self-energy of the process  $X \rightarrow Y$ .

### III. SIMPLIFIED 3 + 1 MODEL AND CONSTRAINTS

In order to illustrate the effect of a heavy neutrino on the triple Higgs coupling we introduce a simplified

model that includes 3 light neutrinos and an extra heavy neutrino. All of them are Dirac fermions and the heavy neutrino couples to the SM particles through its mixing with SM fields. This reproduces the behavior of heavy sterile neutrinos present in seesaw extensions of the SM with approximately conserved lepton number for example [19–21, 24–27]. In the mass basis, the relevant couplings between neutrinos and SM bosons are given by

$$\begin{aligned} \mathcal{L} \ni & -\frac{g_2}{\sqrt{2}} \bar{\ell}_i \not{W}^- B_{ij} P_L n_j + \text{H.c.} \\ & -\frac{g_2}{2\cos\theta_W} \bar{n}_i \not{Z} C_{ij} P_L n_j \\ & -\frac{g_2}{\sqrt{2}M_W} \bar{\ell}_i G^- B_{ij} (m_{\ell_i} P_L - m_{n_j} P_R) n_j + \text{H.c.} \\ & -\frac{g_2}{2M_W} \bar{n}_i C_{ij} H (m_{n_i} P_L + m_{n_j} P_R) n_j \\ & + \frac{ig_2}{2M_W} \bar{n}_i C_{ij} G^0 (-m_{n_i} P_L + m_{n_j} P_R) n_j, \end{aligned} \quad (9)$$

where  $\ell_i$  are the charged leptons of mass  $m_{e/\mu/\tau}$ ,  $n_i$  the Dirac neutrinos of mass  $m_{1\dots 4}$ ,  $g_2$  is the SU(2) coupling constant, and  $B$  and  $C$  are  $4 \times 4$  mixing matrices. In particular,  $B$  and  $C$  are defined as

$$B = R_{34} R_{24} R_{14} \tilde{U}_{PMNS}, \quad (10)$$

$$C_{ij} = \sum_{k=1}^3 B_{ki}^* B_{kj}, \quad (11)$$

with rotation matrices  $R_{34}$ ,  $R_{24}$  and  $R_{14}$  such as

$$R_{14} = \begin{pmatrix} \cos\theta_{14} & 0 & 0 & \sin\theta_{14} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin\theta_{14} & 0 & 0 & \cos\theta_{14} \end{pmatrix}, \quad (12)$$

and  $\tilde{U}_{PMNS}$  the block-diagonal matrix

$$\tilde{U}_{PMNS} = \begin{pmatrix} U_{PMNS} & 0 \\ 0 & 1 \end{pmatrix}, \quad (13)$$

where  $U_{PMNS}$  [30, 31] corresponds to the best-fit point for a normal hierarchy in [32] with  $\delta_{CP} = 0$ . We have also chosen the three light neutrinos to be degenerate with  $m_{n_1/n_2/n_3} = 1$  eV in agreement with the results of the Mainz and Troitsk experiments [33, 34]. Since their small masses translate into small couplings to the Higgs boson, we expect the corrections to the triple Higgs coupling from the light neutrinos to be irrelevant. As a consequence, we have not varied the light neutrino parameters in this work.

Other experimental and theoretical constraints that apply to the heavy sterile neutrino have to be taken into account as well. Direct searches at the LHC are not as constraining [35, 36] as indirect constraints yet and should be even less constraining than claimed in Ref. [35, 36] since the production cross sections were

overestimated according to Ref. [37]. Global fits to various observables have been performed recently [35, 38–40], pointing to electroweak precision observables as the most constraining ones above the Higgs mass. We use here the constraints from the global fit performed in [41, 42],

$$\begin{aligned} B_{e4} &\leq 0.041, \\ B_{\mu 4} &\leq 0.030, \\ B_{\tau 4} &\leq 0.087, \end{aligned} \quad (14)$$

at the 95% C.L. From the theoretical point of view, we require the loop expansion to remain perturbative, applying either a loose (tight) bound of

$$\left( \frac{\max|C_{i4}| g_2 m_{n_4}}{2M_W} \right)^3 < 16\pi (2\pi). \quad (15)$$

The tight bound is roughly equivalent to the bound that comes from a two-loop analysis of the perturbativity of the SM presented in [43, 44]. Using the largest  $C_{i4}$  in agreement with Eq. (14), these translate into upper limits on the heavy neutrino mass of  $m_{n_4} = 14.3$  TeV and  $m_{n_4} = 7.2$  TeV, respectively. However, the decay width of the heavy neutrino grows as  $m_{n_4}^3$ . In order for the quantum state to be a definite particle, we also require  $\Gamma_{n_4} \leq 0.6 m_{n_4}$ , which limits the upper value of  $m_{n_4}$  to approximately 9 TeV for  $B_{\tau 4} = 0.087$ . These limits depend on the values chosen for the  $B_{i4}$  and a decreased mixing increases the upper limit on  $m_{n_4}$ .

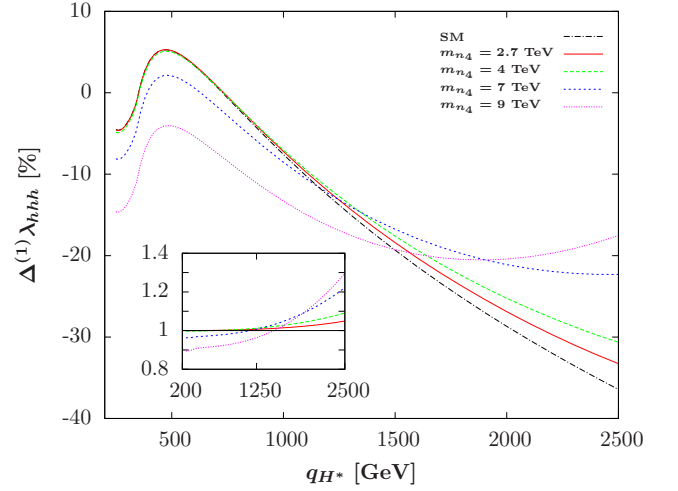


FIG. 1. One-loop corrections to the triple Higgs coupling  $\lambda_{HHH}$  (in %) as a function of the momentum  $q_{H^*}$  of the splitting  $H^*(q_{H^*}) \rightarrow HH$  (in GeV), for several values of the neutrino mass parameter  $m_{n_4}$  and at a constant  $B_{\tau 4} = 0.087$ . The ratio of the genuine BSM contribution to the triple Higgs coupling with respect to the one-loop SM contribution is shown in the insert.

#### IV. RESULTS

For the numerical evaluation of the one-loop corrections, the SM parameters are chosen as

$$\begin{aligned} m_t &= 173.5 \text{ GeV}, \quad M_H = 125 \text{ GeV}, \quad M_W = 80.385 \text{ GeV}, \\ M_Z &= 91.1876 \text{ GeV}, \quad \alpha^{-1} = 127.934. \end{aligned} \quad (16)$$

The new neutrino contributions to the one-loop diagrams of  $\lambda_{HHH}^{(1)}$  are

$$\begin{aligned} \lambda_{HHH}^{(1,\nu)} &= -\frac{\alpha\sqrt{4\pi\alpha}}{32\pi M_W^3 s_W^3} \sum_{j,k,l=1}^4 (C_{jk}^* C_{kl}^* C_{lj}^* + C_{jl}^* C_{lk}^* C_{kj}^*) \left[ m_{n_j}^2 m_{n_k}^2 (4q_{H^*}^2 C_1 + (4M_H^2 + q_{H^*}^2) C_2) \right. \\ &\quad + m_{n_l}^2 (q_{H^*}^2 C_1 (5m_{n_j}^2 + 3m_{n_k}^2) + C_2 (2m_{n_j}^2 (M_H^2 + q_{H^*}^2) + m_{n_k}^2 (2M_H^2 + q_{H^*}^2))) \\ &\quad \left. + 4B_0 (m_{n_k}^2 m_{n_l}^2 + m_{n_l}^2 m_{n_j}^2 + m_{n_j}^2 m_{n_k}^2) + C_0 m_{n_j}^2 (m_{n_l}^2 (q_{H^*}^2 + 4m_{n_j}^2) + m_{n_k}^2 (q_{H^*}^2 + 4m_{n_j}^2 + 8m_{n_l}^2)) \right], \end{aligned} \quad (17)$$

with  $B_0$ ,  $C_0$ ,  $C_1$  and  $C_2$  the scalar and tensor integrals [45, 46]

$$\begin{aligned} B_0 &\equiv B_0(M_H^2, m_{n_k}^2, m_{n_l}^2), \\ C_0 &\equiv C_0(q_{H^*}^2, M_H^2, M_H^2, m_{n_j}^2, m_{n_k}^2, m_{n_l}^2), \\ C_{1/2} &\equiv C_{1/2}(q_{H^*}^2, M_H^2, M_H^2, m_{n_j}^2, m_{n_k}^2, m_{n_l}^2). \end{aligned} \quad (18)$$

This comes in addition to the SM corrections that we have recalculated and found to agree with the literature, see, for example, Ref. [47] and references therein.

In Fig. 1, we present the full one-loop correction, including the neutrinos contribution, as a function of  $q_{H^*}$  where the genuine BSM effects are depicted in the insert as ratio over the SM one-loop result. Here  $q_{H^*}$  is the momentum of the initial off-shell Higgs boson in the splitting  $H(q_{H^*}) \rightarrow HH$ . We have varied  $m_{n_4}$  while keeping  $B_{\tau 4} = 0.087$  fixed at its maximum allowed value given in Eq. (14). The other neutrino mixing terms are zero.

The choice  $m_{n_4} = 2.7$  TeV corresponds to the case where the heavy neutrino effective coupling to the

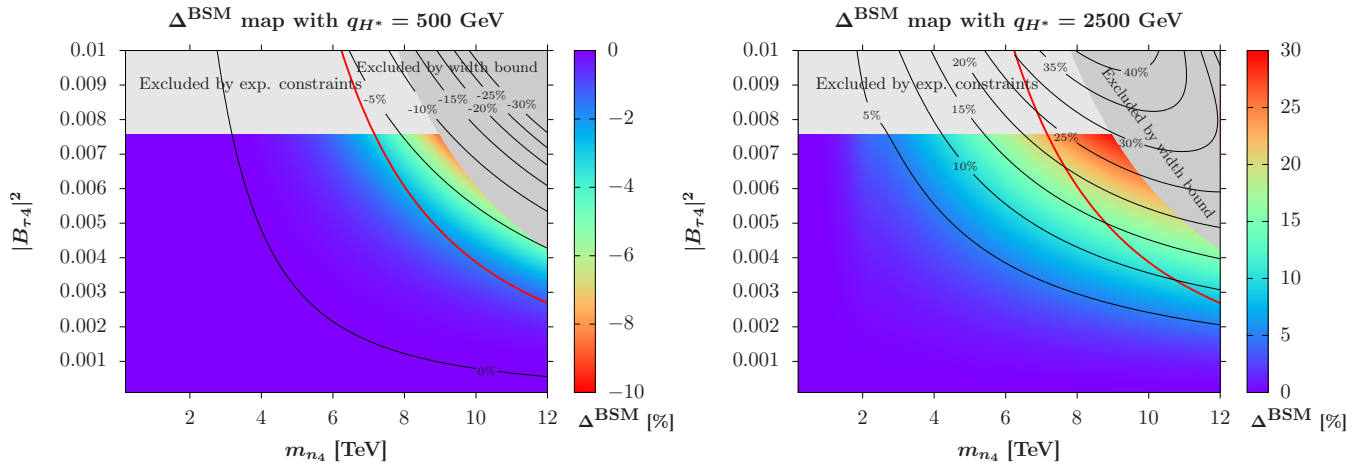


FIG. 2. Contour maps of the neutrino corrections  $\Delta^{\text{BSM}}$  to the triple Higgs coupling  $\lambda_{HHH}$  (in percent) as a function of the two neutrino parameters  $|B_{\tau 4}|^2$  and  $m_{n_4}$  (in TeV), at a fixed off-shell Higgs momentum  $q_{H^*} = 500$  GeV (left) and  $q_{H^*} = 2500$  GeV (right). The other heavy neutrino mixing parameters are set to zero. The light gray area is excluded by the experimental constraints and the darker gray area is excluded from having  $\Gamma_{n_4} > 0.6 m_{n_4}$  while the red line corresponds to the tight perturbativity bound.

Higgs boson is equal to the top-quark Yukawa coupling, i.e.  $\max(C_{i4})m_4 \sim m_t$ , while  $m_{n_4} = 7$  TeV and  $m_{n_4} = 9$  TeV respectively correspond to the tight perturbativity and the width bounds. As can be seen, all scenarios lead to sizable one-loop corrections at low momentum. The largest positive one-loop corrections in the SM are at  $q_{H^*} \simeq 500$  GeV, where the BSM contribution tends to decrease them to  $-3\%$  at  $m_{n_4} = 7$  TeV and  $-9\%$  at  $m_{n_4} = 9$  TeV. The most interesting effects come at larger momentum where the deviation from the SM increases with larger  $m_{n_4}$ , reaching  $+22\%$  at  $q_{H^*} = 2500$  GeV for  $m_{n_4} = 7$  TeV and even  $+30\%$  for  $m_{n_4} = 9$  TeV.

To get an idea of the dependence of the heavy neutrino effect on the model parameters, we have scanned in Fig. 2 the parameter space  $(m_{n_4}, |B_{\tau 4}|^2)$  while fixing the remaining parameters  $B_{e4} = B_{\mu 4} = 0$ .  $\Delta^{\text{BSM}}$  is the percentage correction of the neutrino effects over the one-loop SM triple Higgs coupling defined in Eq. (7). The off-shell Higgs momentum has been fixed to the most interesting values identified in Fig. 1,  $q_{H^*} = 500/2500$  GeV and the experimental and theoretical bounds are included as gray areas. When the heavy neutrino mass and mixing are small, its contribution vanishes as expected. As is seen in Eq. (17), the BSM correction exhibits a strong dependence on the heavy neutrino mass, as terms of order  $\mathcal{O}(m_{n_4}^4)$  and  $\mathcal{O}(m_{n_4}^6)$  arise and they increase  $|\Delta^{\text{BSM}}|$  gradually to reach the maximum allowed deviations  $\Delta^{\text{BSM}} \simeq -10\%$  at  $q_{H^*} = 500$  GeV and  $\Delta^{\text{BSM}} \simeq +30\%$  at  $q_{H^*} = 2500$  GeV. Similar contour maps are found when one trades  $B_{\tau 4}$  against either  $B_{e4}$  or  $B_{\mu 4}$ .

Given the projected sensitivity of around 50% per experiment on the measure of  $\lambda_{HHH}$  at the HL-LHC [48], which translates into a sensitivity of 35% when statistically combining ATLAS and CMS, the effect of the heavy

neutrino may already be probed at the HL-LHC in the case of the maximal value of  $m_{n_4} = 9$  TeV. The International Linear Collider (ILC), one of the future potential electron-positron colliders, at 500 GeV would reach a precision of 27% with  $4 \text{ ab}^{-1}$  while at 1 TeV with  $5 \text{ ab}^{-1}$  the projected sensitivity improves to 10% [49], making the effects clearly visible. At the FCC-hh the effects become important enough to constrain the heavy neutrino mass and mixing, as the projected statistical precision on  $\lambda_{HHH}$  at the FCC-hh with  $3 \text{ ab}^{-1}$  is expected to be 13% per experiment using only the  $b\bar{b}\gamma\gamma$  [50]. Combining the two experiments and using the other search channels, it is reasonable to expect a  $\sim 5\%$  sensitivity which is the target sensitivity of the FCC for this observable. The  $+22\%$  increase predicted at high  $q_{H^*}$  with a conservative perturbativity limit is 4 times the projected sensitivity. Clearly, measuring the neutrino effect or constraining neutrino models in a region hard to probe otherwise becomes possible.

## V. SUMMARY AND OUTLOOK

In this article we have investigated the one-loop effects of a heavy neutrino on the triple Higgs coupling in a simplified model that accounts for the light neutrino masses and mixing and contains one extra heavy neutrino. After taking into account the experimental constraints we have found that the maximum effect can be as large as a 30% increase over the SM one-loop effects, independently of specific flavor structures. The effect is of the order of the projected experimental accuracy on the measure of  $\lambda_{HHH}$  at the HL-LHC, and if a tighter perturbative bound is used on the 3+1 model, it is still possible to have a  $+20\%$  deviation that is 2 times the projected experimental accuracy at the ILC at 1 TeV with  $5 \text{ ab}^{-1}$  and

4 times the one at the FCC-hh with  $3 \text{ ab}^{-1}$ , thus clearly visible. This is the first time the effects of an extended neutrino sector on the triple Higgs coupling have been investigated, demonstrating that it provides a novel way of probing neutrino mass models in a regime otherwise difficult to access. This provides an extra motivation to the experimental measurement of this coupling. It should be noted that the calculation can be extended to all models using the neutrino portal, like dark matter models, and to cases with more heavy neutrinos where the effects can be enhanced. This is confirmed by our preliminary study in the inverse seesaw [51].

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- [1] G. Aad *et al.* (ATLAS Collaboration), Phys.Lett., **B716**, 1 (2012), [arXiv:1207.7214 \[hep-ex\]](#).
  - [2] S. Chatrchyan *et al.* (CMS Collaboration), Phys.Lett., **B716**, 30 (2012), [arXiv:1207.7235 \[hep-ex\]](#).
  - [3] P. W. Higgs, Phys.Lett., **12**, 132 (1964).
  - [4] F. Englert and R. Brout, Phys.Rev.Lett., **13**, 321 (1964).
  - [5] P. W. Higgs, Phys.Rev.Lett., **13**, 508 (1964).
  - [6] G. Guralnik, C. Hagen, and T. Kibble, Phys.Rev.Lett., **13**, 585 (1964).
  - [7] J. M. Cornwall, D. N. Levin, and G. Tiktopoulos, Phys. Rev. Lett., **30**, 1268 (1973), [Erratum: Phys. Rev. Lett.31,572(1973)].
  - [8] C. H. Llewellyn Smith, Phys. Lett., **B46**, 233 (1973).
  - [9] J. Baglio, *Proceedings, 22nd International Workshop on Deep-Inelastic Scattering and Related Subjects (DIS 2014)*, Pos, **DIS2014**, 120 (2014), [arXiv:1407.1045 \[hep-ph\]](#).
  - [10] J. Baglio, A. Djouadi, and J. Quevillon, (2015), [arXiv:1511.07853 \[hep-ph\]](#).
  - [11] Y. Fukuda *et al.* (Super-Kamiokande), Phys. Rev. Lett., **81**, 1562 (1998), [arXiv:hep-ex/9807003 \[hep-ex\]](#).
  - [12] P. Minkowski, Phys. Lett., **B67**, 421 (1977).
  - [13] P. Ramond, in *International Symposium on Fundamentals of Quantum Theory and Quantum Field Theory Palm Coast, Florida, February 25-March 2, 1979* (1979) pp. 265–280, [arXiv:hep-ph/9809459 \[hep-ph\]](#).
  - [14] M. Gell-Mann, P. Ramond, and R. Slansky, *Supergravity Workshop Stony Brook, New York, September 27-28, 1979*, Conf. Proc., **C790927**, 315 (1979), [arXiv:1306.4669 \[hep-th\]](#).
  - [15] T. Yanagida, *Proceedings: Workshop on the Unified Theories and the Baryon Number in the Universe, Tsukuba, Japan, 13-14 Feb 1979*, Conf. Proc., **C7902131**, 95 (1979).
  - [16] R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett., **44**, 912 (1980).
  - [17] J. Schechter and J. W. F. Valle, Phys. Rev., **D22**, 2227 (1980).
  - [18] J. Schechter and J. W. F. Valle, Phys. Rev., **D25**, 774 (1982).
  - [19] R. N. Mohapatra, Phys. Rev. Lett., **56**, 561 (1986).
  - [20] R. N. Mohapatra and J. W. F. Valle, Phys. Rev., **D34**, 1642 (1986).
  - [21] J. Bernab  , A. Santamaria, J. Vidal, A. Mendez, and J. W. F. Valle, Phys. Lett., **B187**, 303 (1987).
  - [22] A. Pilaftsis, Z. Phys., **C55**, 275 (1992), [arXiv:hep-ph/9901206 \[hep-ph\]](#).
  - [23] A. Ilakovac and A. Pilaftsis, Nucl. Phys., **B437**, 491 (1995), [arXiv:hep-ph/9403398 \[hep-ph\]](#).
  - [24] E. K. Akhmedov, M. Lindner, E. Schnapka, and J. W. F. Valle, Phys. Lett., **B368**, 270 (1996), [arXiv:hep-ph/9507275 \[hep-ph\]](#).
  - [25] E. K. Akhmedov, M. Lindner, E. Schnapka, and J. W. F. Valle, Phys. Rev., **D53**, 2752 (1996), [arXiv:hep-ph/9509255 \[hep-ph\]](#).
  - [26] S. M. Barr, Phys. Rev. Lett., **92**, 101601 (2004), [arXiv:hep-ph/0309152 \[hep-ph\]](#).
  - [27] M. Malinsk  , J. C. Rom  o, and J. W. F. Valle, Phys. Rev. Lett., **95**, 161801 (2005), [arXiv:hep-ph/0506296 \[hep-ph\]](#).
  - [28] A. Denner, Fortsch. Phys., **41**, 307 (1993), [arXiv:0709.1075 \[hep-ph\]](#).
  - [29] D. T. Nhung, M. M  hlleitner, J. Streicher, and K. Walz, JHEP, **11**, 181 (2013), [arXiv:1306.3926 \[hep-ph\]](#).
  - [30] B. Pontecorvo, Sov. Phys. JETP, **6**, 429 (1957), [Zh. Eksp. Teor. Fiz.33,549(1957)].
  - [31] Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys., **28**, 870 (1962).
  - [32] M. C. Gonzalez-Garcia, M. Maltoni, and T. Schwetz, JHEP, **11**, 052 (2014), [arXiv:1409.5439 \[hep-ph\]](#).
  - [33] C. Kraus *et al.*, Eur. Phys. J., **C40**, 447 (2005), [arXiv:hep-ex/0412056 \[hep-ex\]](#).



- [34] V. N. Aseev *et al.* (Troitsk), Phys. Rev., **D84**, 112003 (2011), [arXiv:1108.5034 \[hep-ex\]](#).
- [35] F. F. Deppisch, P. S. Bhupal Dev, and A. Pilaftsis, New J. Phys., **17**, 075019 (2015), [arXiv:1502.06541 \[hep-ph\]](#).
- [36] A. Das and N. Okada, Phys. Rev., **D93**, 033003 (2016), [arXiv:1510.04790 \[hep-ph\]](#).
- [37] C. Degrande, O. Mattelaer, R. Ruiz, and J. Turner, (2016), [arXiv:1602.06957 \[hep-ph\]](#).
- [38] S. Antusch and O. Fischer, JHEP, **10**, 094 (2014), [arXiv:1407.6607 \[hep-ph\]](#).
- [39] S. Antusch and O. Fischer, JHEP, **05**, 053 (2015), [arXiv:1502.05915 \[hep-ph\]](#).
- [40] A. de Gouvêa and A. Kobach, Phys. Rev., **D93**, 033005 (2016), [arXiv:1511.00683 \[hep-ph\]](#).
- [41] F. del Aguila, J. de Blas, and M. Pérez-Victoria, Phys. Rev., **D78**, 013010 (2008), [arXiv:0803.4008 \[hep-ph\]](#).
- [42] J. de Blas, *Proceedings, 1st Large Hadron Collider Physics Conference (LHCP 2013)*, EPJ Web Conf., **60**, 19008 (2013), [arXiv:1307.6173 \[hep-ph\]](#).
- [43] L. Durand, P. N. Maher, and K. Riesselmann, Phys. Rev., **D48**, 1084 (1993), [arXiv:hep-ph/9303234 \[hep-ph\]](#).
- [44] U. Nierste and K. Riesselmann, Phys. Rev., **D53**, 6638 (1996), [arXiv:hep-ph/9511407 \[hep-ph\]](#).
- [45] G. 't Hooft and M. J. G. Veltman, Nucl. Phys., **B153**, 365 (1979).
- [46] G. Passarino and M. J. G. Veltman, Nucl. Phys., **B160**, 151 (1979).
- [47] A. Arhrib, R. Benbrik, J. El Falaki, and A. Jueid, JHEP, **12**, 007 (2015), [arXiv:1507.03630 \[hep-ph\]](#).
- [48] V. Khachatryan *et al.* (CMS Collaboration), *Higgs pair production at the High Luminosity LHC*, Tech. Rep. CMS-PAS-FTR-15-002 (CERN, 2015).
- [49] K. Fujii *et al.*, (2015), [arXiv:1506.05992 \[hep-ex\]](#).
- [50] H.-J. He, J. Ren, and W. Yao, Phys. Rev., **D93**, 015003 (2016), [arXiv:1506.03302 \[hep-ph\]](#).
- [51] J. Baglio and C. Weiland, *in preparation*, (2016).